Autonomous Bookkeeping

Workload Transfer, Human Intervention

and Finite-Portfolio Capacity

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Abstract

We study accepted bookkeeping work completed without human intervention. A fixed-portfolio example separates workload ownership from labour efficiency and delivery quality. A compound exception model gives the probability of an unattended close and the moments of residual human effort. We derive a lower bound on the dispersion of that effort and a sufficient finite-portfolio capacity bound. The latter identifies the reserve required when interventions are correlated. The numerical example relates these results to a hypothetical transition from staff execution to autonomous completion.

1. Introduction

Monthly bookkeeping comprises heterogeneous tasks with dependencies and acceptance requirements. Bills must be entered, transactions coded, accounts reconciled and closing records delivered. In this note, work is autonomous when its accepted outcome is reached without human input. Execution, verification and self-recovery belong to the automated workflow; human-required intervention is measured separately. Xara supplies the motivating application.

Task-level autonomy and whole-close autonomy concern different events. The former measures the share of a workload completed independently. The latter asks whether an entire client-month requires any human handling. Average staff effort describes a third quantity. These measures must be distinguished when assessing the transfer of work to an autonomous system.

Section 2 defines workload ownership and historical performance comparisons. Section 3 models unresolved exceptions and repeated handling. Section 4 derives the resulting concentration and capacity bounds. Section 5 evaluates a numerical example using the stipulated portfolio and an explicitly specified probability model.

2. Ownership and performance

Fix the work scope before execution. Let $b_j > 0$ denote item j's historical effort weight, c_j indicate accepted completion, and u_j indicate completion without human input. Define

$$A = \frac{\sum_j b_j c_j u_j}{\sum_j b_j}, \qquad W = \sum_j b_j c_j, \qquad \phi = \frac{H}{W}. \tag{1}$$

Here H is total human effort, including assistance and correction. The operating objective is $A \uparrow$ and $\phi \downarrow$, subject to first-pass acceptance $Q_1 \geq Q_0$ and on-time completion $D_1 \geq D_0$. Unfinished work remains in the denominator of A.

The study portfolio comprises 13 pilot clients, with accepted output evaluated across the historical baseline and pilot observation periods. The numerical inputs are specified in Figure 1 and the accompanying table.

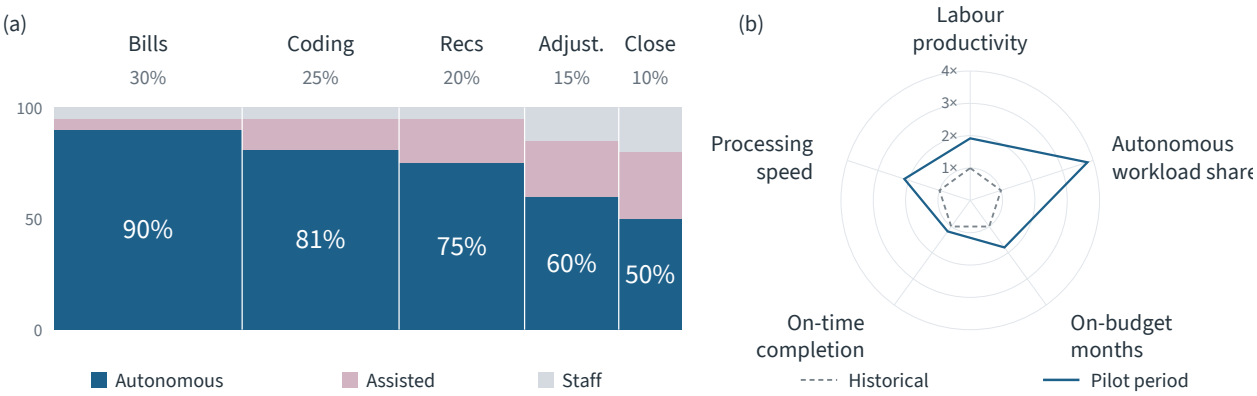


Figure 1. Workload ownership and relative performance. (a) Column width represents historical effort; segment height represents ownership. Weighted shares: 76.25% autonomous, 15.75% assisted, 8.00% staff. (b) Ratios $I_k/100$ from Eq. (2); dashed outline: historical baseline; blue outline: pilot period.

For higher-is-better metric M_k, the historical comparator is its monthly mean over the same cohort:

$$\overline{M}_{0k} = \frac{1}{T} \sum_{t=-T}^{-1} M_{tk}, \qquad I_k = 100 \frac{M_{1k}}{\overline{M}_{0k}}. \tag{2}$$

Metric	Historical mean	Pilot period	Index I_k
Client-months per staff-hour	0.056	0.108	191.9
Autonomous workload share	20.0%	76.3%	381.3
On-budget months	55.6%	100.0%	180.0
On-time completion	80.0%	95.0%	118.8
Completion speed (inverse days)	1/15	1/7	214.3

3. Whole-close autonomy and residual effort

An autonomous close has no compulsory human stage. The autonomous system executes, verifies and recovers; only unresolved cases require human intervention. Self-recovery contributes no human effort to the model.

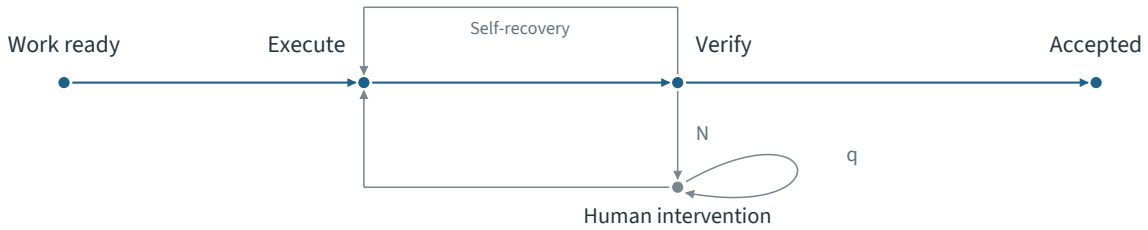


Figure 2. Autonomous execution with conditional human intervention. The autonomous system executes, verifies and recovers. N counts episodes requiring human input; q is the probability of another human handling round. For $N = 0$, the close requires no human input.

Condition on eventual accepted completion. For one client-month, let

$$H = \sum_{j=1}^{N} Y_j, \qquad Y_j = \sum_{\ell=1}^{G_j} X_{j\ell}, \tag{3}$$
$$\mathbb{P}(G_j = g) = (1-q)q^{g-1}, \qquad g \geq 1.$$

Assume $\mathbb{E}N = \lambda$, $\mathrm{Var}(N) = \nu$, $0 \leq q < 1$, and positive handling times X with mean μ and variance σ^2. The G_j are iid, the $X_{j\ell}$ are iid, and these sequences and N are mutually independent, conditional on the client class. An empty sum is zero.

Proposition 1 (Effort moments and unattended completion)**.** *Writing $m = \mathbb{E}H$ and $v = \mathrm{Var}(H)$,*

$$m = \frac{\lambda\mu}{1-q}, \tag{4}$$
$$v = \frac{\lambda\sigma^2}{1-q} + \frac{(\lambda q + \nu)\mu^2}{(1-q)^2}, \tag{5}$$
$$\alpha := \mathbb{P}(H = 0) = \mathbb{P}(N = 0). \tag{6}$$

For $N \sim \mathrm{Poisson}(\lambda)$, $\alpha = e^{-\lambda}$.

Proof. The geometric law gives $\mathbb{E}G = (1-q)^{-1}$ and $\mathrm{Var}(G) = q(1-q)^{-2}$. Conditioning on G yields $\mathbb{E}Y = \mu/(1-q)$ and $\mathrm{Var}(Y) = \sigma^2/(1-q) + q\mu^2/(1-q)^2$. Conditioning on N gives $m = \lambda\mathbb{E}Y$ and $v = \lambda\,\mathrm{Var}(Y) + \nu(\mathbb{E}Y)^2$. Positivity of X makes $H = 0$ equivalent to $N = 0$. □

4. Residual effort and portfolio capacity

Proposition 2 (Concentration of residual human effort)**.** *For any nonnegative H with $0 < \alpha < 1$ and finite second moment,*

$$\mathbb{E}[H \mid H > 0] = \frac{m}{1-\alpha}, \qquad \frac{v}{m^2} \geq \frac{\alpha}{1-\alpha}. \tag{7}$$

Proof. Since $m = \mathbb{E}[H\mathbf{1}_{\{H>0\}}]$, Cauchy–Schwarz gives $m^2 \leq (1-\alpha)\mathbb{E}H^2$. Rearrangement yields the variance bound. Equality holds when positive human effort is constant. □

At $\alpha = 0.8$ and $m = 5$ hours, affected closes average 25 human hours and $\mathrm{CV}(H) \geq 2$. Task autonomy A and whole-close autonomy α therefore describe different units. Likewise, m alone does not identify α: for any $\lambda > 0$, choosing $\mu = m(1-q)/\lambda$ preserves m while changing $e^{-\lambda}$.

Let $S_n = \sum_{i=1}^{n} H_i$, let B be the staff-hour budget within a common closing window, and take $0 < \varepsilon < 1$. Define

$$C_\varepsilon(B) = \sup\{n \in \mathbb{N}_0 : \mathbb{P}(S_n \leq B) \geq 1 - \varepsilon\}. \tag{8}$$

This is a human-workload feasibility criterion. Calendar completion also depends on document release times, agent capacity and task precedence.

Proposition 3 (A sufficient capacity bound)**.** *Suppose the client-months have common moments and pairwise covariance*

$$\mathbb{E}H_i = m, \qquad \mathrm{Var}(H_i) = v, \qquad \mathrm{Cov}(H_i, H_j) = \rho v \quad (i \neq j),$$

where $0 \leq \rho \leq 1$. Then $\mathbb{P}(S_n \leq B) \geq 1 - \varepsilon$ whenever

$$\boxed{nm + \sqrt{\frac{1-\varepsilon}{\varepsilon}\, v\,[n + \rho n(n-1)]} \;\leq B.} \tag{9}$$

Proof. The sum has mean nm and variance $v[n + \rho n(n-1)]$. Cantelli's inequality bounds $\mathbb{P}(S_n - nm \geq t)$ by $\mathrm{Var}(S_n)/(\mathrm{Var}(S_n) + t^2)$. For positive variance, set $t = B - nm > 0$ and solve for a bound of ε. Zero variance gives the result directly. □

The sufficient per-client reserve converges to

$$\frac{B_{\mathrm{suff}}(n)}{n} - m \longrightarrow \sqrt{\frac{1-\varepsilon}{\varepsilon}\, v\rho}. \tag{10}$$

For $\rho = 0$, the reserve vanishes as n grows. Fixed positive correlation leaves a nonzero limit.

5. Numerical illustration and reproducibility

Set $N \sim \text{Poisson}(0.25)$, $X \sim \text{Exp}(\text{mean } 15\text{h})$ and $q = 0.25$. These assumptions give $m = 5\text{h}$, $v = 200\text{h}^2$ and $\alpha = 77.88\%$; only the mean is matched to the 200-hour, 40-client fixture. Geometric sums of these exponentials are exponential with mean $\theta = \mu/(1-q)$. For independent clients,

$$\mathbb{P}(S_n \le B) = e^{-n\lambda} \sum_{k=0}^{\infty} \frac{(n\lambda)^k}{k!} F_{\Gamma(k,\theta)}(B), \tag{11}$$

where θ is a scale parameter and $\Gamma(0, \theta)$ is a point mass at zero.

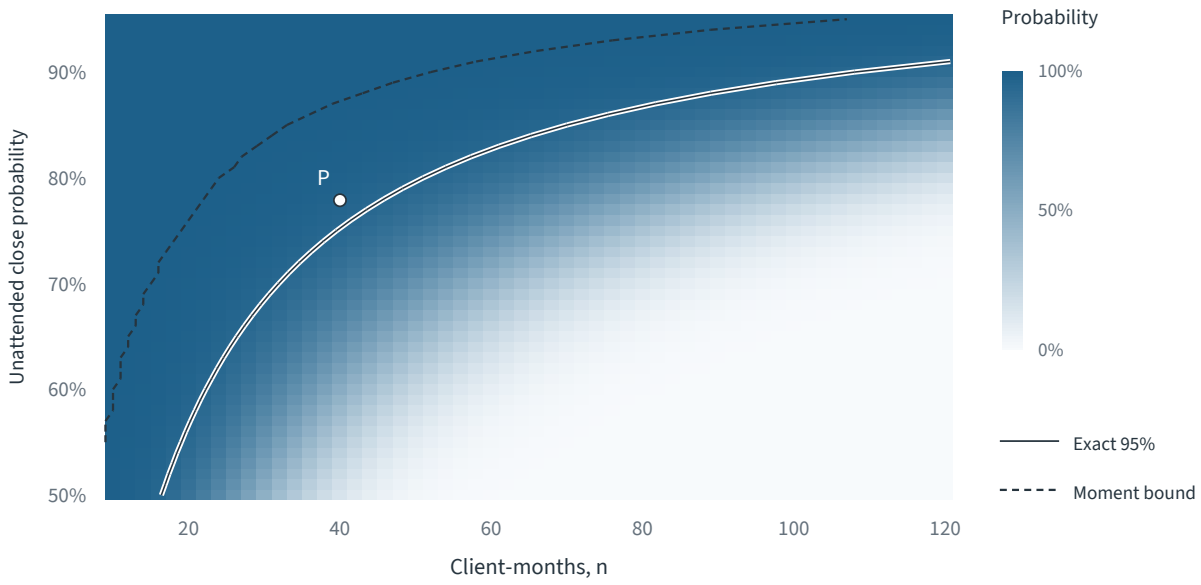


Figure 3. Capacity under a human-effort budget. Colour represents $\Pr(S_n \le B)$ for $B = 400$ hours. Varying α varies $\lambda = -\log \alpha$; $\mu = 15$ hours and $q = 0.25$ are fixed. Solid: exact 95% boundary. Dashed: moment-based 95% guarantee with $\rho = 0$. Point P denotes 40 client-months.

At the marked point, $\mathbb{P}(S_{40} \le 400) = 97.42\%$ and $C_{0.05}(400) = 45$. The moment bound certifies 22 clients; with $\rho = 0.02$, it certifies 18. These are respectively a distribution-specific capacity and conservative guarantees, not interchangeable estimates.

Reproducibility. The source bundle contains all test inputs, vector figures and numerical code. Equation (11) is evaluated through independent Poisson probabilities, with omitted mixture mass below 10^{-11}. A separate 120,000-draw check verifies the effort moments and zero-input probability.